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MATHEMATICAL MODELS OF PARTLY PROTECTED FISH POPULATIONS [1]

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Keywords: population modeling; restricted fishing; delay equations.

Abstract: We propose mathematical model to examine management strategies associated with fully protected marine reserves. We consider two linked populations in adjacent areas, where one is exposed to fishing and the other is not.

Introduction.

Traditional methods of controlling fishing include: restricting the fishing season, restricting the number or biomass of fish captured, restricting the size of fish captured, restricting the places available for fishing. The last option includes the establishment of marine reserves, i. e. areas permanently closed to all fishing. Reserves could help to maintain healthy ecosystems with natural levels of species and habitat diversity. Therefore, theoretical studies of harvesting strategies are of great interest and have attracted attention of both scientists and managers.

In this work we use mathematical models to examine management strategies associated with fully protected marine reserves. We consider two linked populations in adjacent areas, where one is exposed to fishing and the other is not. Our models allow us to explore the influence of a reserve on the levels of stock biomass and catch. The key issue will also be an user-friendly software for numerical analysis of effective harvesting strategies in the context of protected marine populations.

The fishing industry in Mozambique.

Fishing is an important area of the Mozambican economy, and since l973 production and marketing of saltwater fish, shrimp, and shellfish have increased steadily. Mozambique's offshore waters contain tuna, mackerel, sardines, and anchovies but are best known for the shrimp (prawns) that are an important export commodity. A long-term, sustainable management of the nation's fishing resources based on scientifically approved guidelines is therefore of vital importance. A quantitative understanding of this kind can only be achieved by combining modern mathematical modeling methods with a profound knowledge of the biology of the populations.

The main mathematical model.

We consider two regions that have two habitat areas A and B, with a fish population that dispersing between the two areas. Assume that fishing takes place only in region B, with region A established as an marine reserve area or no-fishing zone.

From the point of view of fishery managers, the existence of globally attractive solutions is very important in order to plan harvesting strategies and sustain the fishing grounds. The life-history migration characteristics and management parameters, such as the fishing rates and the areas allocated to the reserve and fishing zones, may influence the dynamics of the system.

The following system of delay differential equations is studied

$$\frac{dx_1}{dt} = x_1(t)\,[1 - x_1^{\gamma_1}(t)] - a_1 x_1(t) + a_2 x_2(t)$$
$$\frac{dx_2}{dt} = x_2(t)\,[1 - x_2^{\gamma_2}(t-h)] + a_1 x_1(t) - a_2 x_2(t) - h(x_2(t)),$$

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where $\pm a_i x_i(t)$ are migration terms and $h(x_2(t))$ is the harvesting term. The delay in the second equation is due to the maturity period of the stock which is to be harvested. As in many other biological models we exploit more general power law nonlinearities instead of logistic ones.

Analysis of this model and its comparison with the non-delay model was done using Mathematica.

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Аннотация: Предлагается математическая модель развития двух соседствующих популяций одного вида рыб в ситуации, когда одна из рассматриваемых популяций находится в зоне, запрещённой для рыболовства.

Ключевые слова: популяционная модель; уравнения с запаздыванием.

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